The Finite Element Method

The purpose of this assignment is to implement a simple finite element code written in MATLAB to solve the two-dimensional heat conduction problem introduced in the lectures

$$\begin{aligned} q_{i,i} &= 0 \quad \text{in } \Omega \\ u &= g \quad \text{on } \Gamma_g \\ -q_i n_i &= h \quad \text{on } \Gamma_h \end{aligned}$$

in which $q_i = -\kappa_{ij} u_{,i}$ where κ is the material conductivity, u is the temperature, and q is the heat flux, and to investigate some of the properties of the solution.

You should submit a full well-presented report which addresses the points below. The report should be no longer than 10 pages in length. You should include all your MATLAB code in an appendix which does not form part of the page count. The submitted file should be entitled `yourlastname_yourCID.pdf`. You should include a signed declaration that the report contains solely your own work.

Assignment

1. Clearly describe your finite element implementation of the heat conduction problem. Your implementation should allow the domain to be discretized using both four-node and eight-node quadrilateral finite elements.

   [30%]

2. Consider the two-dimensional trapezoidal plate, shown in Figure 1 on the following page, which has Dirichlet and Neumann boundary conditions as shown. The temperature specified on boundary Γ^g is 300 K. Your aim is to determine the required constant heat flux h on Γ^h which will result in the mean temperature across the whole domain having a value of 250 K.

   Assume an *isotropic* conductivity $\kappa = 20.0$ Wm^{-1}K^{-1} throughout. The plate thickness is 0.1 m and all other dimensions are as shown.

   (a) Investigate and discuss the relationship between the solution convergence and the mesh density for the two element types.

   [50%]

   (b) Investigate and discuss the effect of nodal numbering on the bandwith of the assembled K matrix.

   [20%]

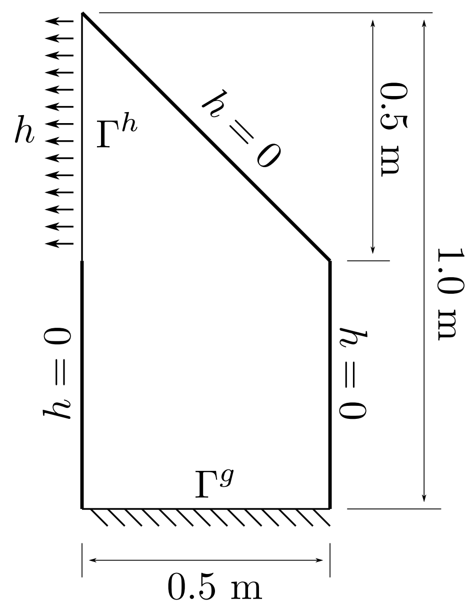

Figure 1: Trapezoidal domain for the heat conduction problem